NOTATION

Π(porosity; δ_{cs}) thickness of capillary structure; L) lenth of the heat pipe; R_p) internal radius of the heat pipe; r) radius of the vapor space; R_r) radius of rotation; V_ℓ) volume charged with liquid; V_s) volume of liquid in the stream; V_{cs}) volume of liquid in the capillary structure; V_d) volume of liquid drawn out; D_{ef}) effective diameter of the pores; P_h) hydrostatic pressure; P_c) capillary head pressure; ρ) density of heat transfer agent; g) acceleration of free fall; ω) angular velocity of rotation; σ) surface tension; Θ) wetting angle; H) height of rise of heat transfer agent; Π') degree of filling of the capillary structure.

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NUMERICAL STUDY OF THE PROBLEM OF UNSTEADY HEAT TRANSFER IN A CASED-WELL–BED SYSTEM

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A solution is presented for an unsteady axisymmetric heat-conduction problem connected with the development of the theory of the geothermal logging of cased wells. The solution is used as a basis for reaching several practical conclusions regarding the feasibility of conducting geothermal logging in cased wells.

Thermal monitoring techniques employing an active source have recently come into wide use in flaw detection. To optimize measurement conditions and gain a better understanding of the physics of the pertinent phenomena, investigators have solved a number of two- and three-dimensional problems of unsteady heat transfer in nonuniform laminated media. Examples of such problems are those studied in [1, 2]. In the present investigation, we solve a similar problem connected with the theory of geothermal logging. The results may prove useful in refining the theory and optimizing logging conditions. The solution of this problem is also of interest in regard to developing methods of detecting flaws on tubes of heat exchangers and fuel elements in nuclear reactors.

Mathematically modeling the processes which take place in geothermal logging makes it possible to study the effect of the parameters of the probe (power and concentration of the source, velocity and length of the probe), the thermal properties of the fluid and the walls of the well, and geometric factors on the distribution of the temperature field and heat sources in the well–bed system. Here, we study the effect of the casing string and cement ring on unsteady heat transfer in the well–bed system and we evaluate the effect of the velocity and length of the probe on the feasibility of performing geothermal logging in cased wells.

We will examine a cylindrical region (Fig. 1) containing a well of radius R_1 filled with a fluid (liquid or gas) having a thermal conductivity λ_1, heat capacity c_1, and density ρ_1. It is assumed that the well has been cased with a string having the thermal properties λ_2, c_2, and ρ_2. The external radius of the string is R_2. The casing string has been cemented, the cement ring being represented as a hollow cylinder with an external radius R_3

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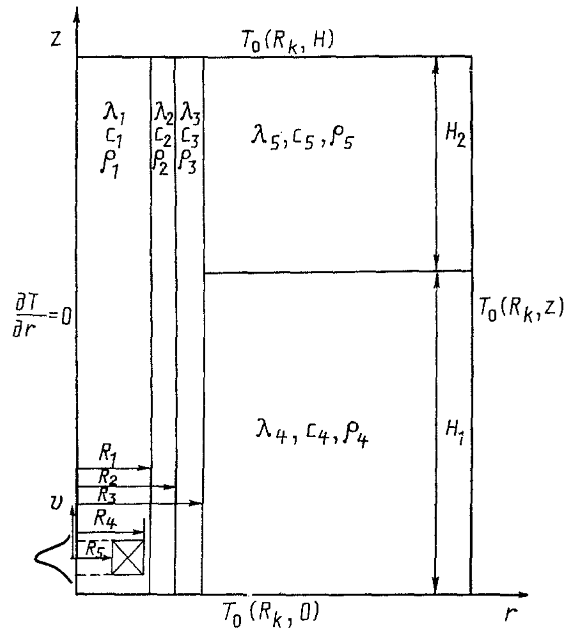

Fig. 1. Model of cased-well—bed system.

and the thermal properties λ_3, c_3, ρ_3. Two beds of capacities H_1, H_2 and different properties (λ_4, c_4, ρ_4, λ_5, c_5, ρ_5) are separated by a horizontal boundary. The well contains an annular movable heat source of unit power Q with the dimensions R_4, R_5.

Unsteady heat transfer in such an axisymmetric region is described by the heat-conduction equation. In a cylindrical coordinate system, this equation has the form

$$c\rho \frac{\partial T}{\partial t} = \frac{I}{r} \frac{\partial}{\partial r}\left(r\lambda \frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) + Q(r, z, t). \tag{1}$$

Let the power of the moving source be represented in the following form ($Q_0(r)$ being a function of the radius, allowing us to vary the dimensions of the source): $Q = Q_0(r) \exp(-k[(z - z_0) - vt]^2)$.

The following conditions of continuity of temperature and heat flux must be satisfied on all horizontal and vertical boundaries:

$$T_i = T_{i+1}, \quad \lambda_i \left(\frac{\partial T}{\partial x}\right)_i = \lambda_{i+1} \left(\frac{\partial T}{\partial x}\right)_{i+1}. \tag{2}$$

Since the problem is being solved numerically on a computer, we will examine a finite region represented by a cylinder. The height ($H = H_1 + H_2$) and radius (R_k) of the cylinder are assumed to be such as to have minimal effect on the solution of the problem. On the external boundary of the cylinder at $r = R_k$ (usually taken equal to several meters) we assign a temperature distribution corresponding to the undisturbed natural temperature field of the Earth $T_0(R_k, z)$.

On the lower and upper boundaries of the cylinder, we can assign the condition of constancy of the temperature or heat flux:

$$T(r, z = 0) = T_0(R_k, 0), \quad T(r, z = H) = T_0(R_k, H) \quad \text{or} \quad q(r, z = 0) = q(r, z = H) = \text{const}. \tag{3}$$

On the axis of the well, it is necessary to satisfy the symmetry condition

$$\frac{\partial T}{\partial r}(r = 0, z) = 0. \tag{4}$$

As the initial conditions in the given region, we can either take the temperature corresponding to the stationary distribution in the porous medium or we can assume that it is everywhere equal to zero:

$$T(r, z) = 0. \tag{5}$$

We will use the finite-differences method [4] (locally unidimensional computing scheme) to numerically solve the problem (Eqs. (1-5)). To do this, we replace the space (r, z, t) by its grid analog ω_k (ih_1, jk_2, τn), and we replace the continuous function T(r, z, t) by the grid function T^n_{ij}. The chosen grid has a uniform mesh with respect to z equal to h_2 = const. Its mesh is also uniform (h_{10} = const) in the radial direction (inside the well, at $r \leq R_1$); at $r > R_1$, the mesh of the grid is nonuniform $h_{1i} = h_{10}q^{i-1}$ (where q = const).

We will write Eq. (1) in finite-difference form

$$(c\rho)_{ij}\frac{T^{n+1}_{ij}-T^n_{ij}}{\tau}=\frac{1}{r_i\bar{h}_i}\left[(r\lambda)_{i+\frac{1}{2}j}\left(\frac{T^{n+1}_{i+1j}-T^{n+1}_{ij}}{h_{1i+1}}\right)-\right.$$

$$\left.-(r\lambda)_{i-\frac{1}{2}j}\left(\frac{T^{n+1}_{ij}-T^{n+1}_{i-1j}}{h_{1i}}\right)\right]+\frac{1}{h_2}\left[\lambda_{ij+\frac{1}{2}}\left(\frac{T^{n+1}_{ij+1}-T^{n+1}_{ij}}{h_2}\right)-\right. \quad (6)$$

$$\left.-\lambda_{ij-\frac{1}{2}}\left(\frac{T^{n+1}_{ij}-T^{n+1}_{ij-1}}{h_2}\right)\right]+Q^n_{ij}.$$

Proceeding in accordance with the locally unidimensional scheme being employed [4] and introducing a fractional time step (n + 1/2) as we split Eq. (6) into two parts, we reduce this equation to the following system of algebraic equations:

$$A_{ij}T^{n+\frac{1}{2}}_{ij-1}-B_{ij}T^{n+\frac{1}{2}}_{ij}+C_{ij}T^{n+\frac{1}{2}}_{ij+1}=-D_{ij}, \quad (7)$$

$$E_{ij}T^{n+1}_{i-1j}-F_{ij}T^{n+1}_{ij}+G_{ij}T^{n+1}_{i+1j}=-I_{ij},$$

where i = 1,..., I; j = 1,..., J;

$$A_{ij}=\frac{\lambda_{ij-\frac{1}{2}}}{h_2^2};\quad C_{ij}=\frac{\lambda_{ij+\frac{1}{2}}}{h_2^2};\quad B_{ij}=A_{ij}+C_{ij}+\frac{(\rho c)_{ij}}{\tau};$$

$$D_{ij}=\frac{(\rho c)_{ij}}{\tau}T^n_{ij};\quad E_{ij}=\frac{(r\lambda)_{i-\frac{1}{2}j}}{r_1h_{1i}\bar{h}_i};\quad \bar{h}_i=\frac{h_{1i}+h_{1i+1}}{2};$$

$$G_{ij}=\frac{(r\lambda)_{i+\frac{1}{2}j}}{r_ih_{1i+1}\bar{h}_1};\quad F_{ij}=E_{ij}+G_{ij}+\frac{(\rho c)_{ij}}{\tau};$$

$$I_{ij}=\frac{(\rho c)_{ij}T^{n+\frac{1}{2}}_{ij}}{\tau}+Q^n_{ij}.$$

System (7) was solved (successively) by the trial-run method [4]. Use of this approach makes it possible to easily find the solution of a system of algebraic equations whose matrix contains nontrivial coefficients only on three diagonals. In this case, using the recursion formula

$$T^{n+\frac{1}{2}}_{ij}=\alpha_{ij+1}T^{n+\frac{1}{2}}_{ij+1}+\beta_{ij+1},\quad i=1,\ \ldots,\ I;\quad j=1,\ \ldots,\ J, \quad (8)$$

and the left boundary condition (T = const or $\partial T/\partial r = 0$), we can find a sequence of correction factors $\{\alpha_j\}$, $\{\beta_j\}$. Then using the right boundary condition ($T = T(R_k, z)$), we employ Eq. (8) to execute the reverse trial run and determine the sought function $T^{n+\frac{1}{2}}_{ij}$. We subsequently proceed in the same manner and execute the forward and reverse trial runs in another direction (with respect to i), thus determining the value of T^{n+1}_{ij} throughout the region. The process is continued until we obtain the distribution of T^n_{ij} throughout the interval of interest to us.

The absolute stability of the computing scheme follows from satisfaction of the conditions [4]

$$A_{ij}\geqslant 0;\ C_{ij}\geqslant 0;\ A_{ij}+C_{ij}\leqslant B_{ij};\ E_{ij}\geqslant 0;\ G_{ij}\geqslant 0;\ E_{ij}+G_{ij}\leqslant F_{ij}.$$

The error of the approximation of the solution of problem (1-5) can be evaluated (with q = 1) by breaking up the step and in our case it is ≤15%.

The calculations were performed on an ES-1065 computer. The grid contained I × J = 51 × 101 nodes.

We used the above model, realized in the form of programs written for the computer, to perform numerous computations for different system parameters corresponding to actual conditions.

Let us present some of the results obtained for the model (Fig. 1) using the following parameters: H_1 = 10 m; H_2 = 5 m; R_1 = 0.1 m; R_2 = 0.11 m, R_3 = 0.12 m; λ_1 = 0.6 W/(m·K); c_1 = 4200 J/(kg·K); ρ_1 = 1000 kg/m^3 in the case when the well was filled with fluid, and λ_1 = 0.026 W/(m·K); c_1 = 1000 J/(kg·K); ρ_1 = 1.2 kg/m^3 in the case when the well was filled with air; the parameters λ_2 = 30 W/(m·K); c_2 = 800 W/(kg·K); ρ_2 = 7200 kg/m^3 correspond to the thermal properties of the metal string; λ_3 = 1.8 W/(m·K); c_3 = 1000 J/(kg·K); ρ_3 = 2600 kg/m^3 correspond to the thermal properties of the cement. The themal parameters of the beds for the given model were taken as follows: λ_4 = 2 W/(m·K); λ_5 = 4 W/(m·K); $c_4 = c_5$ = 1000 J/(kg·K); $\rho_4 = \rho_5$ = 2800 kg/m^3. The parameters of the heat source were Q_0 = 10^6 W/(m^3); $R_4 = R_1$; R_5 = 0.05 m; k = 100; z_0 = 0.3 m; d = $R_3 - R_2$ = 0.01 m.

Figure 2a shows the relations obtained from the solution of the problem along with the corresponding curves recorded using a temperature probe provided with an active source and having the length L = 4 m. Curve 1 corresponds to a simple temperature probe (T(z)), while curves 2 and 3 correspond to differential probes measuring the first ($\Delta T = (T(z + \ell) - T(z - \ell))/2\ell$) and second ($\Delta^2 T = (T(z + \ell) - 2T(z) + T(z - \ell))/\ell^2$) differences in the temperature distribution along the wall of the well after the passage of a heat source pressed against it when the well was filled with air (ℓ = 0.1 m is the distance between sensors). It follows from Fig. 2a that use of the method of thermometry with an active source in the given case makes it possible to find the boundary between beds in a cased well, assuming that the length and velocity of the probe have been properly chosen. The chosen length and velocity should be such as to ensure that the period of time between the passage of the source and the passage of the sensors is long enough to allow the heat to travel to the boundary between the beds. Here, it should be noted that the use of three sensors to measure the temperature on the well wall makes it possible to instrumentally obtain the analog of the second difference of the temperature distribution Δ^2T, which is more sensitive to the presence of a horizontal boundary between beds than T(z) or ΔT(z). This illustrates the preferability of using differential thermometric methods when solving problems connected with well-logging.

However, the wells that are usually studied are filled with liquid, which makes their

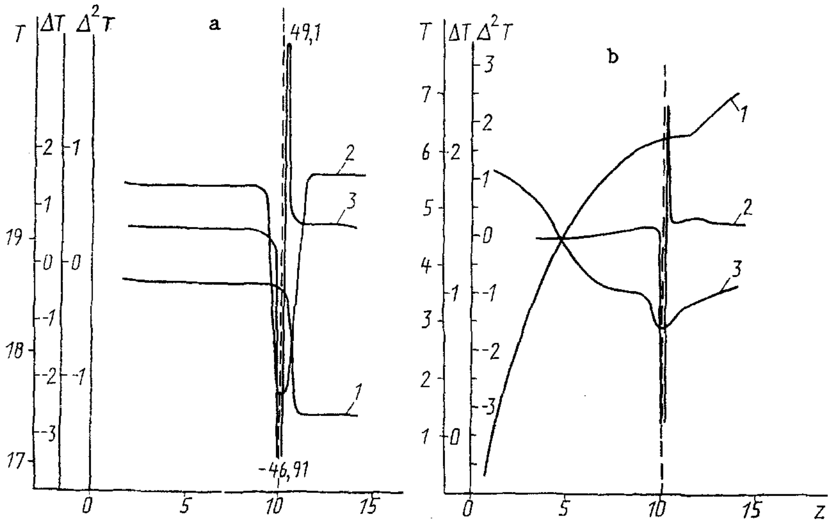

Fig. 2. Distribution of the quantities T, ΔT, Δ^2T, corresponding to the curves recorded with temperature probes of the length L = 4 m, v = 0.025 m/sec: 1) temperature probe which measured T(z); 2, 3) differential probes which measured ΔT(z) and Δ^2T(z), respectively; a) well filled with air; b) well filled with liquid. T, °C; z, m.

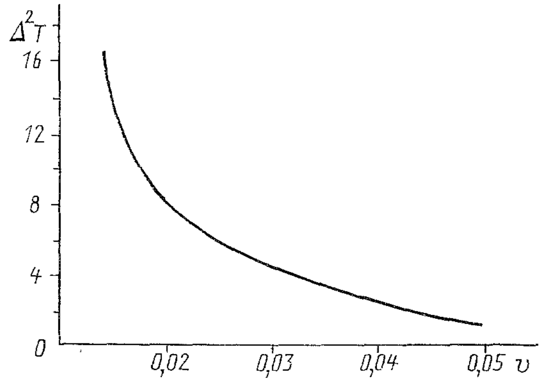

Fig. 3. Dependence of the parameter $(\Delta^2T)_{max}$ on probe velocity v with d = 0.01 m (filler – water). v, m/sec.

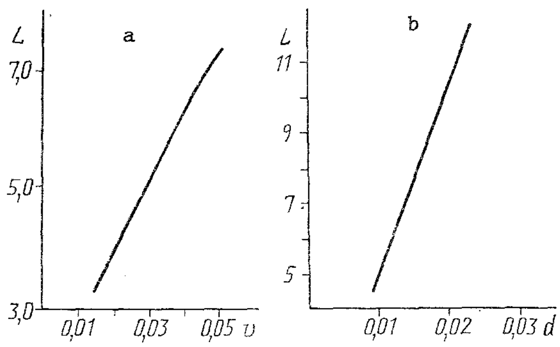

Fig. 4. Dependence of the length of the probe permitting measurement of the maximum value $(\Delta^2T)_{max}$ on its velocity v (d = 0.01 m) (a) and on the thickness of the cement stone d (v = 0.025 m/sec) (b). L, d, m.

thermal logging considerably more complicated. Estimates show that the thermal probe (source and detector) should be clamped together, since the existence of even a small gap between the probe and the wall of the well will significantly reduce its sensitivity and increase the inertia of the method. Such an adverse effect can be explained physically as a deterioration in the conditions of heat transfer between the probe and the wall due to the occurrence of convective heat transfer in the gap.

The presence of liquid in a well also prolongs the process of heating of the wall to a quasisteady regime, since the liquid has a higher thermal conductivity and heat capacity than air (Fig. 2). In connection with this, traditional thermometric techniques do not (especially in the case of a cemented metal string) give good results unless the measurement parameters are optimized beforehand. Figure 2b shows curves analogous to those shown in Fig. 2a for the case when the well is filled with liquid. It follows from these calculations that there is a certain optimum probe length (L_{opt}) which, for the given values of the parameters of the model, makes it possible to measure the maximum value of Δ^2T.

Here, as in the case of a well filled with air, differential thermometric methods are more informative (Fig. 2b). An increase in probe velocity is accompanied by changes not only in the measurable level of a quantity (the sensitivity of the method) (Fig. 3) but also in the distance between the probe and the sensor (detector). This makes it possible to measure the maximum signal connected with detection of the horizontal boundary between the metal string and the cement ring. Figure 4a shows the change in probe lenght which yields the maximum measurable signal for Δ^2T in relation to the velocity of the proble. Besides clamping the probe to the wall of the well, the effectiveness of geothermal logging with an active source can be enhanced by making the surface of the probe that contacts with the wall out of a material which is a good thaml insulator. In this case, the process of heat transfer between the wall and the liquid in the well becomes close to adiabatic. The latter in turn markedly improves the sensitivity of the method and speeds up attainment of a quasisteady temperature dsitribution along the wall. Figure 4b shows the dependence of the length of a probe which permits measuremetn of the maximum value of (Δ^2T) on the thickness of the cement (d) with a constant velocity v = 0.025 m/sec. This thickness is determined by the condition of the wall or the design of the well and may differ in different cases.

It should be noted that the relations presented here change quantitatively in relation to the thermal properties of the rock comprising the bed, the material of the casing, and the cement. However, their behavior remains qualitatively the same within a broad range of values for the parameters of the model.

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